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# Non-Linear Evolution and High Energy Diffractive Production

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## Abstract

The ratio of the diffractive production to the total cross section in DIS is computed as a function of the produced mass. The analysis is based on the solution to the non-linear evolution equation for the diffraction dissociation in DIS.

The obtained ratios almost do not depend on the central mass energy in agreement with the HERA experimental data. This independence is argued to be a consequence of the scaling phenomena displayed by the cross sections.

As a weakness point a significant discrepancy between the data and the obtained results is found in the absolute values of the ratios. Several explanatory reasons are discussed.

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# 1 Introduction

One of the most intriguing experimental observations in HERA is in the energy independence of the ratio between the cross section of single diffractive dissociation and the total DIS cross section [1] (see Fig.1). The widely used saturation model of Golec-Biernat and Wusthoff (GW)

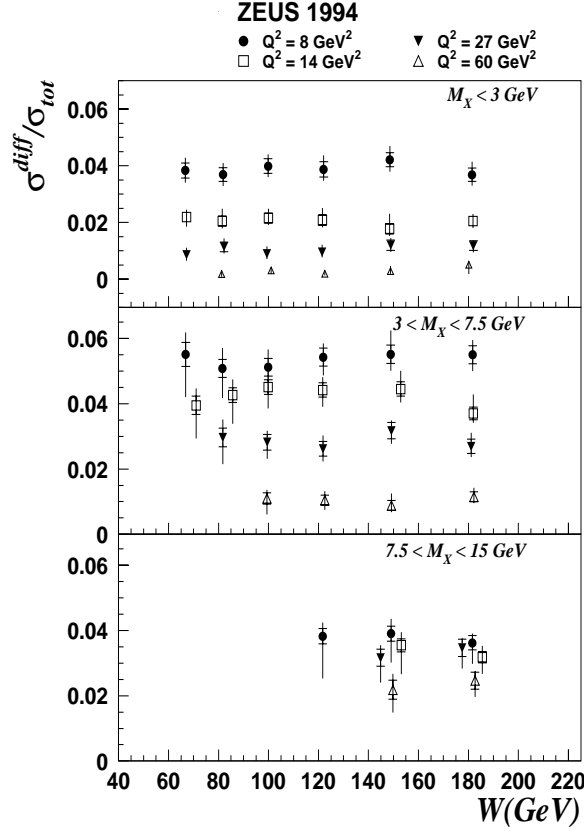


Figure 1: *Experimental data for the ratio  $\sigma_{diff}/\sigma_{tot}$  taken from Ref. [1].*

quite successfully reproduces this data [2]. It was conjectured by Kovchegov and McLerran that the effects of the parton density saturation occurring at high energies are responsible for this independence [3]. Using the unitarity constraint they related the diffraction cross section ( $\sigma_{diff}$ ) to the total cross section ( $\sigma_{tot}$ ) in DIS of  $q\bar{q}$  pair with a target

$$R \equiv \frac{\sigma_{diff}}{\sigma_{tot}} = \frac{\int d^2b \int dz \int d^2r_{\perp} P^{\gamma*}(z, r_{\perp}; Q^2) N^2(r_{\perp}, x; b)}{2 \int d^2b \int dz \int d^2r_{\perp} P^{\gamma*}(z, r_{\perp}; Q^2) N(r_{\perp}, x; b)} . \quad (1.1)$$

The function  $N(r_{\perp}, x; b) = \text{Im} a_{\text{dipole}}^{\text{el}}(r_{\perp}, x; b)$ , where  $a_{\text{dipole}}^{\text{el}}$  is the amplitude of the elastic scattering for the dipole of the size  $r_{\perp}$  and rapidity  $Y \equiv \ln(1/x)$  scattered at impact parameter  $b$ . The Bjorken  $x$  is related to the central mass energy  $W$  via  $x = Q^2/W^2$ .  $P^{\gamma*}(z, r_{\perp}; Q^2)$  is the

probability to find a quark-antiquark pair with the size  $r_\perp$  inside the virtual photon [4, 5]:

$$\begin{aligned} P^{\gamma^*}(z, r_\perp; Q^2) &= \frac{\alpha_{em} N_c}{2\pi^2} \sum_f Z_f^2 \sum_{\lambda_1, \lambda_2} \{ |\Psi_T|^2 + |\Psi_L|^2 \} \\ &= \frac{\alpha_{em} N_c}{2\pi^2} \sum_f Z_f^2 \{ (z^2 + (1-z)^2) a^2 K_1^2(a r_\perp) + 4 Q^2 z^2 (1-z)^2 K_0^2(a r_\perp) \}, \end{aligned} \quad (1.2)$$

where in the quark massless limit  $a^2 = z(1-z)Q^2$  and  $\Psi_{T,L}$  stand for the  $q\bar{q}$  wave functions of transversely and longitudinally polarized photons.

For the amplitude  $N$  a non-linear evolution equation was derived [6, 7, 8, 9, 10, 11, 12]. This equation has been studied both analytically [12, 13] and numerically [11, 14, 15, 16].

Even with inclusion of an extra gluon emission Eq. (1.1) fails to describe correctly the experimental data of Fig. 1 [17, 18, 19, 20]. However, Eq. (1.1) can be used as initial condition to a further evolution.

Similarly to the total cross section we introduce the cross section for diffractive production with the rapidity gap larger than given  $Y_0 \equiv \ln(1/x_0)$ :

$$\sigma_{diff}(x, x_0, Q^2) = \int d^2b \int d^2r_\perp \int dz P^{\gamma^*}(z, r_\perp; Q^2) N^D(r_\perp, x, x_0; b). \quad (1.3)$$

The function  $N^D$  is the amplitude of the diffractive production induced by the dipole with the size  $r_\perp$  and rapidity gap larger than given ( $Y_0$ ). The minimal rapidity gap  $Y_0$  can be kinematically related to the maximal diffractively produced mass  $x_0 = (Q^2 + M^2)/W^2$ . The amplitude  $N^D$  is a subject to a non-linear evolution equation derived for the diffraction dissociation processes in Ref. [21] and recently rederived in Ref. [19]:

$$\begin{aligned} N^D(\mathbf{x}_{01}, Y, Y_0; b) &= N^2(\mathbf{x}_{01}, Y_0; b) e^{-\frac{4C_F \alpha_S}{\pi} \ln(\frac{\mathbf{x}_{01}}{\rho})(Y-Y_0)} + \frac{C_F \alpha_S}{\pi^2} \int_{Y_0}^Y dy e^{-\frac{4C_F \alpha_S}{\pi} \ln(\frac{\mathbf{x}_{01}}{\rho})(Y-y)} \times \\ &\int_\rho d^2\mathbf{x}_2 \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{02}^2 \mathbf{x}_{12}^2} [ 2 N^D(\mathbf{x}_{02}, y, Y_0; \mathbf{b} - \frac{1}{2}\mathbf{x}_{12}) + N^D(\mathbf{x}_{02}, y, Y_0; \mathbf{b} - \frac{1}{2}\mathbf{x}_{12}) N^D(\mathbf{x}_{12}, y, Y_0; \mathbf{b} - \frac{1}{2}\mathbf{x}_{02}) \\ &- 4 N^D(\mathbf{x}_{02}, y, Y_0; \mathbf{b} - \frac{1}{2}\mathbf{x}_{12}) N(\mathbf{x}_{12}, y; \mathbf{b} - \frac{1}{2}\mathbf{x}_{02}) + 2 N(\mathbf{x}_{02}, y; \mathbf{b} - \frac{1}{2}\mathbf{x}_{12}) N(\mathbf{x}_{12}, y; \mathbf{b} - \frac{1}{2}\mathbf{x}_{02}) ]. \end{aligned} \quad (1.4)$$

The equation (1.4) describes a diffraction process initiated by dipole of the size  $\mathbf{x}_{01}$  which subsequently dissociates to two dipoles with the sizes  $\mathbf{x}_{02}$  and  $\mathbf{x}_{12}$ . The rapidity  $Y$  is defined as  $Y = \ln(1/x)$ . First numerical solution of this equation was recently obtained in Ref. [22]. At the energy equal to the minimal energy gap diffraction is purely given by the elastic scattering as it is stated in Eq. (1.1):

$$N^D(r_\perp, x_0, x_0; b) = N^2(r_\perp, x_0; b). \quad (1.5)$$

In the present letter we compute the ratio  $\sigma_{diff}/\sigma_{tot}$  in mass bins. For the function  $N$  and  $N^D$  we use the numerical solutions obtained in Refs. [14, 22].

The letter is organized as follows. In the next section (2) we compute the  $\sigma_{diff}/\sigma_{tot}$  ratio. To this goal we first study the  $b$ -dependence of the amplitude  $N^D$ . Discussion of the results is presented in section 3. We conclude in the last section (4).

## 2 $\sigma_{diff}/\sigma_{tot}$

We assume the following  $b$ -dependence of  $N^D$ :

$$N^D(r_\perp, x, x_0; b) = (1 - e^{-\kappa^D(x, x_0, r_\perp) S(b)})^2, \quad (2.6)$$

with

$$\kappa^D(x, x_0, r_\perp) = -\ln(1 - \sqrt{\tilde{N}^D(r_\perp, x, x_0)}). \quad (2.7)$$

$\tilde{N}^D(r_\perp, x, x_0)$  computed in Ref. [22] represents a solution of the same equation (1.4) but with no dependence on the forth variable. The initial conditions for the function  $\tilde{N}^D(r_\perp, x, x_0)$  are set at  $b = 0$ . In order to estimate the accuracy of the ansatz (2.6) the non-linear equation (1.4) was solved for several values of  $b$  with the only assumption  $r_\perp \ll b$ . The comparison with the ansatz is shown in Fig. 2.

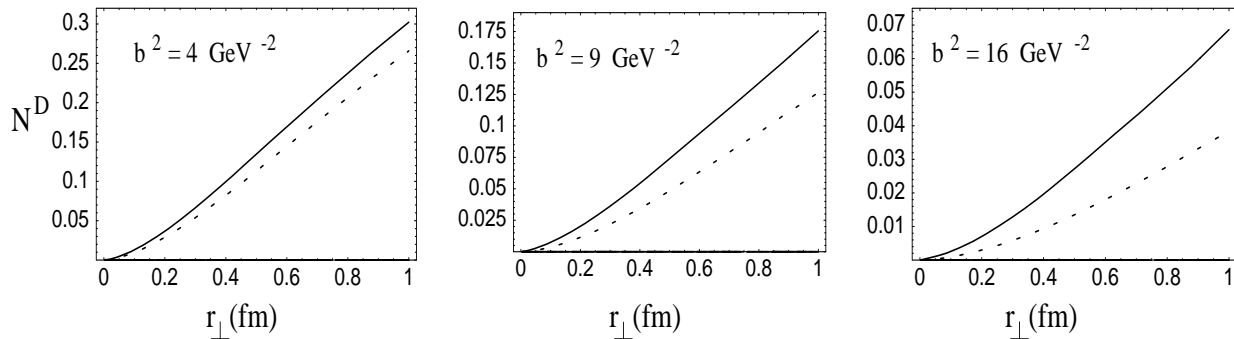


Figure 2: The comparison between the ansatz (2.6)(dashed line) and the true  $b$ -dependence (solid line). The curves are plotted as a function of distance at fixed  $x = 10^{-3}$ .

The ansatz (2.6) underestimates significantly the correct  $b$ -dependence of the amplitude and the mismatch grows with  $b$ . Similar underestimation was obtained for the function  $N$  in Ref. [14] and it can be naturally explained [15]. It is important to note, however, that the mismatch of the function  $N^D$  is significantly larger than the one of the function  $N$ . In the final computation of the ratio this fact leads to underestimation of the ratio especially for smaller  $Q^2$ .

$\sigma_{diff}(x, x_0, Q^2)$  is the cross section for the diffractive production of all masses below given  $M^2 = Q^2(x_0 - x)/x$ . Hence the result for a mass bin can be obtained as a difference between two cross sections corresponding to largest and smallest masses in the bin. Fig. 3 presents the  $R = \sigma_{diff}/\sigma_{tot}$  which is a main result of this letter. From Fig. 3 the ratio  $R$  is observed to be practically independent on the energy  $W$ . This result captures the main feature of the experimental data

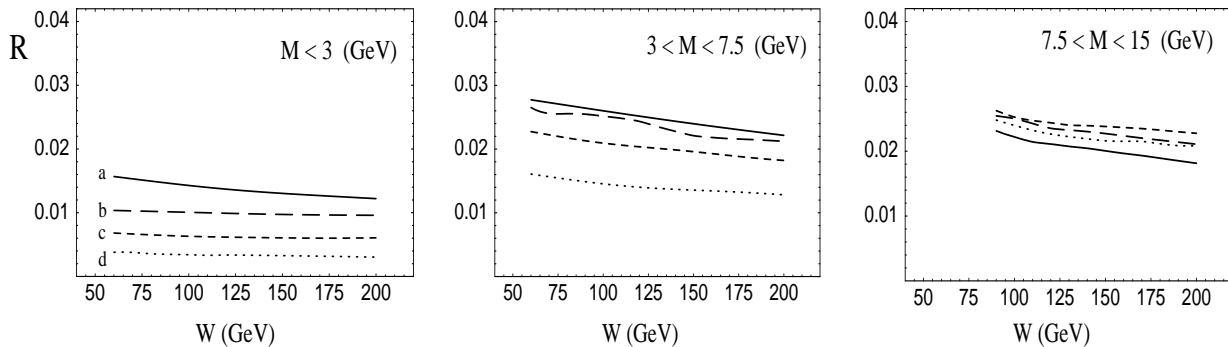


Figure 3: The ratio  $\sigma_{diff}/\sigma_{tot}$  as a function of  $W$ .  $a$  -  $Q^2 = 8 \text{ GeV}^2$ ,  $b$  -  $Q^2 = 14 \text{ GeV}^2$ ,  $c$  -  $Q^2 = 27 \text{ GeV}^2$ , and  $d$  -  $Q^2 = 60 \text{ GeV}^2$ .

(Fig. 1). However the absolute values of the ratios are not reproduced correctly. There are several reasons for this discrepancy which are listed below.

- Due to numerical limitations the  $b$ -dependences of both functions  $N$  and  $N^D$  were simplified and both total and diffractive cross sections were underestimated. However, as was argued above the corresponding errors are not fully canceled in the ratio. More correct treatment of the  $b$ -dependence is likely to enhance the ratio at relatively small  $Q^2$ .
- Both the non-linear evolution equations used in the analysis are valid at very low  $x$ . Moreover, they do not incorporate the correct DGLAP kernel at high  $Q^2$ . The experimental data of Fig. 1 covers kinematic domain where these equations are expected to gain corrections due to DGLAP kernel [14, 23].
- The experimental data (Fig. 1) includes target excitations which are not accounted by the evolution equations. These excitations could in principal reach up to 30% of the diffractive production [17].

The above sources of the uncertainty may potentially change the ratios significantly. Nevertheless we believe that their approximate energy independence would persist in any case. In our opinion, this independence is rather fundamental and related to the scaling phenomena. We will discuss the issue in the next section.

### 3 Discussion

In this section we will argue that the energy independence of the  $\sigma_{diff}/\sigma_{tot}$  ratio can be traced back to the scaling property displayed by the amplitudes  $N$  and  $N^D$  and to the fact that both saturation scales depend on  $x$  with the very same power  $\lambda$  [22].

Both the amplitudes  $N$  and  $N^D$  were discovered to display the remarkable scaling phenomena [24, 22]. Namely,

$$N(r_\perp, x; b) = N(\tau; b); \quad \tau \equiv r_\perp Q_s(x); \quad Q_s(x) = Q_{s0} x^{-\lambda}; \quad \lambda = 0.35 \pm 0.04. \quad (3.8)$$

$$\begin{aligned} N^D(r_\perp, x, x_0; b) &= N(\tau^D; b); \quad \tau^D \equiv r_\perp Q_s^D(x, x_0); \\ Q_s^D(x, x_0) &= Q_{s0}^D(x_0) x^{-\lambda}; \quad \lambda = 0.37 \pm 0.04. \end{aligned} \quad (3.9)$$

The function  $Q_{s0}^D(x_0)$  has a very weak dependence on  $x_0$ . With a quite good accuracy the scaling (3.8, 3.9) was found for all  $x$  below  $x = 10^{-2}$  [24, 22].

Assuming (3.8, 3.9) to be exact property, we can plug the amplitudes into the cross section. As a result, the ratio  $R$  is given by the following expression:

$$R = \frac{Q_s^{D^2}(x, x_0^h) f^D(Q/Q_s^D) - Q_s^{D^2}(x, x_0^l) f^D(Q/Q_s^D)}{Q_s^2(x) f^{tot}(Q/Q_s)}. \quad (3.10)$$

In (3.10)  $x_0^{h,l}$  correspond to high and low masses in a given mass bin. The functions  $f^{tot}$  and  $f^D$  are obtained as a result of the dipole degree of freedom integrations. For the sake of transparency we use the small  $z$  approximation to simplify the wave function integration [4, 5] :

$$\int d^2 r_\perp \int dz P^{\gamma*}(z, r_\perp; Q^2) \rightarrow const \times \int_{4/Q^2} \frac{d^2 r_\perp}{Q^2 r_\perp^4}. \quad (3.11)$$

Using Eq. (3.11) one can obtain the following expressions for  $\sigma_{tot}$  and  $\sigma_{diff}$ :

$$\sigma_{tot} = const \times \int_{4/Q^2} \frac{d^2 r_\perp}{Q^2 r_\perp^4} \int d^2 b N(r_\perp, x; b) = const \tau^2 \int_\tau \frac{d\tau'}{\tau'^3} \int d^2 b N(\tau'; b); \quad (3.12)$$

$$\sigma_{diff} = const \times \int_{4/Q^2} \frac{d^2 r_\perp}{Q^2 r_\perp^4} \int d^2 b N^D(r_\perp, x, x_0; b) = const \tau^{D^2} \int_{\tau^D} \frac{d\tau'^D}{\tau'^{D^3}} \int d^2 b N^D(\tau'^D; b).$$

These equations show that the main contribution in integration over  $\tau$  stems from the region of small  $\tau$  (small dipole sizes). It was shown in Refs. [11, 14, 15, 16, 22, 24] that both  $N(r_\perp, x; b)$  and  $N^D(r_\perp, x, x_0; b)$  at low  $x$  display scaling properties even at short distances where  $N \propto r_\perp^2 Q_s^2(x)$  and  $N^D \propto (r_\perp^2 Q_s^{D^2}(x, x_0))^2$ . Substituting these estimates in Eq. (3.12) one can see that

$$\sigma_{tot} \propto S \tau^2 \ln(\tau) + Const; \quad (3.13)$$

$$\sigma_{diff} \propto S \tau^{D^2} Const, \quad (3.14)$$

with  $S$  standing for the target transverse area.

Consequently, the main power dependence on  $x$  (or  $W$ ) comes from the saturation scales  $Q_s$  and  $Q_s^D$ , which cancels in the ratio. As a result, at most logarithmic dependence could be expected for the ratio.

In our opinion, the scaling property is a fundamental block in explaining the energy independence of the ratio  $R$ . So successful GW saturation model has this scaling built in [2]. To conclude the discussion it is important to note that the scaling (3.8) was discovered in the experimental data on the structure function  $F_2$  [25]. Hence any possible corrections to our analysis mentioned in the previous section are unlikely to spoil the scaling phenomena.

## 4 Conclusions

The letter presents our attempt to reproduce the experimental data (Fig. 1) on  $\sigma_{diff}/\sigma_{tot}$  ratio as a function of the produced mass. In particular focus is the energy independence of the ratios, which is well established experimentally [1].

The analysis is carried on a basis of the non-linear evolution equations derived for the total DIS production in Ref. [9, 10] and for the diffractive production in Ref. [21]. The numerical solutions of these equations used for the analysis were obtained in Refs. [14, 22].

Though our results (Fig. 3) fail to reproduce correctly the experimental data, they successfully reproduce the desired energy independence of the ratios. This independence is explained by relating it to the scaling phenomena which are argued to be a fundamental property of DIS at low  $x$  starting from  $x \approx 10^{-2}$ . These scaling phenomena are found to hold approximately even at short distances ( $r_\perp \ll 1/Q_s$ ) which give dominant contributions to the computed cross sections.

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